MA-101/1841

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B.Tech. (Semester-I)Examination–2013

Mathematics-I

Time: Three Hours

Maximum Marks: **100**

Note: Attempt questions from all the sections.

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Section–A

(Short Answer Type Questions)

Note: Attempt any ten questions. Each question carries 4 marks.

(4x10=40)

1. If $y = e^{\tan^{-1}x}$ prove that
$(1+x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$

2. Trace $y^2(a^2+x^2) = x^2(a^2-x^2)$

3. Expand $sin(xy)$ in powers of $(x-1)$ and $(y-\pi/2)$ as far as the terms of second degree.

4. Discuss the maximum and minimum of
$x^2+y^2+6x+12$

5. Use the method of the Lagrange's multipliers to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $x^2/_{a^2} + y^2/_{b^2} + z^2/_{c^2} = 1$

6. The power P required topropel a steamer of length 'l' at a speed 'u' is given by $P = \lambda u^3 l^3$ where λ is constant. If u is increased by 3% & l is decreased by 1% find the corresponding increase in 'P'

7. Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 3 & 42 \\ 2 & -1 & 32 \\ 3 & -5 & 22 \\ 6 & -3 & 86 \end{bmatrix}.$$

8. Is the system of vectors
$X_1 = (2,2,1)^T,\ X_2 = (1,3,1)^T,$
$X_3 = (1,2,2)^T$ are linearly dependent

9. If $A = \begin{bmatrix} 2+3i & 1-2i & 2+4i \\ 3-4i & 4+3i & 2-6i \\ 5 & 5+6i & 3 \end{bmatrix}$
Find A^{θ} matrix

10. Evaluate $\int_0^2 \int_1^{e^x} dx\, dy$ by charging the order of integration.

11. Find the volume of the solid bounded by the parabolic $y^2 + z^2 = 4x$and the plane $x = 5$.

12. Prove that $\lceil n \lceil 1 - n = \pi / sinn\pi (o < n < 1)$

13. Find the rate of change of $\phi = xyz$ in the direction normal to the surface $x^2y + y^2x + yz^2 = 3$ at the point (1,1,1).

14. Find the divergence of $\vec{V}$ at (2,-1,1) where $\bar{V} = (xyz)\hat{\imath} + (3x^2y)\hat{\jmath} + (xz^2 - y^2z)K$

15. Prove that $\bar{F}$ is irrational
Where$\bar{F} = (y^2 - z^2 + 3yz - 3x)\hat{\imath} + (3xz + 2xy)\hat{\jmath}$
$+(3xy - 2xy + 2z)\hat{K}$

Section–B

(Long Answer Type Questions)

Note: Attempt any three questions. Each question carries 20 marks. **(20x3=60)**

1. Verify Euler's theorem for $u = x^2 \tan^{-1}(y/x) - y^2 \tan^{-1}(x/y)$ and also prove that $\partial^2 u/\partial x dy = (x^2 - y^2)/(x^2 + y^2)$

$\frac{\partial xyz}{\partial}$

2. If $x = r\, sin\theta cos\phi$
   $y = r sin\theta sin\,\phi$
   $z = r cos\,\theta$
   Show that $\partial(x, y, z)/\partial(r, \theta, \phi) = r^2 sin\theta.$

3. Test for consistency the following system of equations & if consistent solve
   $x_1 + 2x_2 - x_3 = 3$
   $3x_1 - x_2 + 2x_3 = 1$
   $2x_1 - 2x_2 + 3x_3 = 2$
   $x_1 - x_2 + x_3 = -1$

4. Find the characteristics equation of

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

   Verify Cayley Hamilton theorem and hence find A^{-1}.

5. Evaluate $\int\int\int x^2yz\, dx\, dy\, dz$ throughout the volume bounded by the planes
   $x = 0, y = 0, z = 0 x/a + y/b + z/c = 1.$

6. Verify divergence theorem, given that
   $\bar{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$and S is the surface of the cube bounded by the planes
   $x = 0, x = 1, y = 0, y = 1\ z = 0, z = 1.$

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