

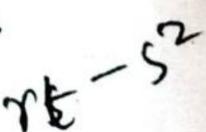
MA-101/1841

B. Tech. (Semester-I)Examination-2013 Mathematics-I

Time: Three Hours

Maximum Marks: 100

Note: Attempt questions from all the sections.



Section-A

(Short Answer Type Questions)

Note: Attempt any ten questions. Each question carries
4 marks.

(4x10=40)

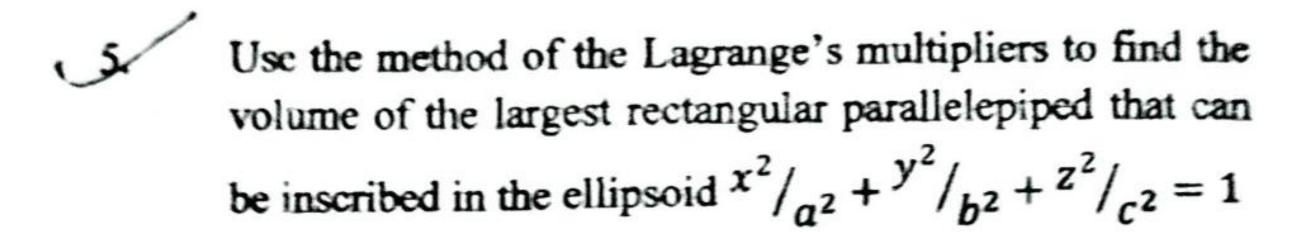
If
$$y = e^{\tan^{-1}x}$$
 prove that
$$(1+x^2)y_{n+2} + [2(n+1)x-1]y_{n+1} + n(n+1)y_n = 0$$

Trace
$$y^2(a^2 + x^2) = x^2(a^2 - x^2)$$

Expand sin(xy) in powers of (x-1) and (y-1/2) as far as the terms of second degree.

Discuss the maximum and minimum of
$$x^2 + y^2 + 6x + 12$$

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6. The power P required topropel a steamer of length 'l' at a speed 'u' is given by $P = \lambda u^3 l^3$ where λ is constant. If u is increased by 3% & 1 is decreased by 1% find the corresponding increase in 'P'

Find the rank of the matrix
$$A = \begin{bmatrix} 1 & 3 & 42 \\ 2 & -1 & 32 \\ 3 & -5 & 22 \\ 6 & -3 & 86 \end{bmatrix}$$

8. Is the system of vectors $X_1 = (2,2,1)^T, X_2 = (1,3,1)^T,$ $X_3 = (1,2,2)^T \text{ are linearly dependent}$

If
$$A = \begin{bmatrix} 2+3i & 1-2i & 2+4i \\ 3-4i & 4+3i & 2-6i \\ 5 & 5+6i & 3 \end{bmatrix}$$

Find A^{θ} matrix

10. Evaluate $\int_0^2 \int_1^{e^x} dx dy$ by charging the order of integration.

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- Find the volume of the solid bounded by the parabolic $y^2 + z^2 = 4x$ and the plane x = 5.
- 12. Prove that $[n(1-n=\pi/sinn\pi(o < n < 1)$
- Find the rate of change of $\phi = xyz$ in the direction normal to the surface $x^2y + y^2x + yz^2 = 3$ at the point (1,1,1).
- Find the divergence of \vec{V} at (2,-1,1) where $\vec{V} = (xyz)\hat{\imath} + (3x^2y)\hat{\jmath} + (xz^2 y^2z)K$

Prove that
$$\overline{F}$$
 is irrational

Where $\overline{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath}$
 $+(3xy - 2xy + 2z)\hat{K}$

Section-B

(Long Answer Type Questions)

Note: Attempt any three questions. Each question carries 20 marks. (20x3=60)

Verify Euler's theorem for $u = x^2 \tan^{-1}(y/x) - y^2 \tan^{-1}(x/y) \text{ and also prove}$ that $\frac{\partial^2 \phi}{\partial x dy} = (x^2 - y^2)/(x^2 + y^2)$

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If
$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$

Show that $\partial(x,y,z)/\partial(r,\theta,\phi)=r^2sin\theta$.

Test for consistency the following system of equations & if consistent solve

$$x_1 + 2x_2 - x_3 = 3$$

$$3x_1 - x_2 + 2x_3 = 1$$

$$2x_1 - 2x_2 + 3x_3 = 2$$

$$x_1 - x_2 + x_3 = -1$$

Find the characteristics equation of $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Verify Cayley Hamilton theorem and hence find A^{-1} .

- Evaluate $\iiint x^2yz \, dx \, dy \, dz$ throughout the volume bounded by the planes x = 0, y = 0, z = 0x/a + y/b + z/c = 1.
- Verify divergence theorem, given that $\bar{F} = 4xz\hat{\imath} y^2\hat{\jmath} + yz\hat{k}$ and S is the surface of the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.

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